Mathematics

Q.1 Let A be a $p \times q$ $(p < q)$ matrix with rank p. Then:

a) For every b in R^p, $AX = b$ has a unique solution.
b) For every b in R^p, $AX = b$ has a solution but not unique.
c) There exists $b \in R^p$ for which $AX = b$ has no solution.
d) None of these

Q.2 Laplace Transformation of $J_0(t)$ is:

a) $\frac{1}{\sqrt{1-s^2}}$
b) $\frac{1}{\sqrt{1+s^2}}$
c) $\frac{1}{\sqrt[3]{1-s^2}}$
d) $\frac{1}{\sqrt[3]{1+s^2}}$

where $J_0(t)$ is Bessel's function of order 0.

Q.3 Let $A = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 1 & 2 \\ 2 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix}$. Then the rank of A is equal to:

a) 1
b) 2
c) 3
d) 4

Q.4 If the derivative of the function

$$f(x) = \begin{cases} ax^2 + b, & x < -1 \\ bx^2 + ax + 4, & x \geq -1 \end{cases}$$

is everywhere continuous, then

a) $a = 2, b = 3$
b) $a = 3, b = 2$
c) $a = -2, b = -3$
d) $a = -3, b = -2$

Q.5 Matrix $A_\alpha = \begin{bmatrix} \alpha & \alpha - 1 \\ \alpha - 1 & \alpha \end{bmatrix}, \alpha \epsilon N$, then the value of $|A_1| + |A_2| + \cdots + |A_{300}| =$

a) $(300)^2$
b) $(299)^2$
c) $(301)^2$
d) None of these

Q.6 The root of the equation $x^2 - 5x + 1 = 0$ between 0 and 1, obtained by using 2 iterations of bisection method, is:

a) 0.25
b) 0.50
c) 0.75
d) 0.65

Q.7 If there are n independent trials, p and q the probability of success and failure respectively, then the probability of exactly r successes is

a) q^n
b) ${}^nC_r\, q^n p^r$
c) ${}^nC_r\, q^{n-r}\, p^r$
d) ${}^nC_r\, p^{n-r} q^r$

Q.8 The median of the variables $x + 4, x - \frac{7}{2}, x - \frac{5}{2}, x - 3, x - 2, x + \frac{1}{2}, x - \frac{1}{2}, x + 5$ $(x > 0)$ is:

a) $x - 3$
b) $x - 2$
c) $x + \frac{5}{4}$
d) $x - \frac{5}{4}$

Q.9 If coeff. of correlation r_{xy} between two variates x and y is 0.6, $\text{cov}(x, y) = 4.8$, $\sigma_x^2 = 9$, then $\sigma_y =$

a) 8/9
b) 5/8
c) 8/3

d) None of these

Q.10 The imaginary part of the principal value of 5^{4+i} is:

a) $625\ \sin(\ln 5)$
b) $125\ \sin(\ln 5)$
c) $25\ \sin(\ln 5)$
d) $5\ \sin(\ln 5)$

Q.11 The differential equation $\frac{du}{dt} = A(a-u)(b-u)$, along with the condition $u(0) = 0$ is:

a) $\frac{b(a-u)}{a(b-u)} = e^{A(a-b)t}$
b) $\frac{b(a+u)}{a(b+u)} = e^{A(a-b)t}$
c) $\frac{b(a-t)}{a(b-t)} = e^{A(a-b)u}$
d) $\frac{b(a+t)}{a(b+t)} = e^{A(a-b)u}$

Q.12 The function $f(z) = z^2$ maps the first quadrant onto:

a) Itself
b) 3rd quadrant
c) Upper half plane
d) Lower half plane

Q.13 If $f(z) = z^2$, then it:

a) Has an essential singularity at $z = \infty$.
b) Has a pole of order 2 at $z = \infty$.
c) Has a pole of order 2 at $z = 0$.
d) Is analytic at $z = \infty$.

Q.14 Let C be a circle $|z| = 3$ oriented anti-clockwise, then

$$\oint_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz =$$

a) 0
b) $4\pi i$
c) $-2\pi i$
d) $2\pi i$

Q.15 Consider the system of equations

$$\begin{bmatrix} 5 & 2 & 1 \\ 1 & 4 & 2 \\ 1 & 2 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 15 \\ 20 \end{bmatrix}$$

with the initial guess of the solution $\left[x_1^{(0)}, x_2^{(0)}, x_3^{(0)}\right]^T = [0,0,0]^T$, the approximate value of the solution $\left[x_1^{(2)}, x_2^{(2)}, x_3^{(2)}\right]^T$ after two iterations by the Jacobi's Iteration Method is:

a) $[2.4, 4, 3.75\ ]^T$
b) $[2.4, 3.75, 4]^T$
c) $[1.15, 0.1, 2.02]^T$
d) $[0.1, 1.15, 2.02]^T$

Q.16 For the function

$$f(z) = \frac{z - \sin z}{z^3} \quad (z \neq 0); f(0) = 0$$

the point $z = 0$ is:

a) A pole of order 3
b) A pole of order 2
c) An essential singularity
d) A removable singularity

Q.17 Let L be a line segment from (1,1,1) to (3,3,3). Then

$$\int_L 2xy^2z^2\, dx + 2yx^2z^2\, dy + 2zx^2y^2\, dz =$$

a) 26
b) 28
c) 730
d) 728

Q.18 Laplace Transformation of $(t-1)^2 u(t-1)$ is:

a) $\dfrac{2e^{-s}}{s^3}$

b) $\dfrac{e^{-(s-1)}}{s^3}$

c) $\dfrac{2e^{-s}}{(s-1)^3}$

d) $\dfrac{2e^{-(s-1)}}{s^3}$

where $u(t)$ is the unit step function.

Q.19 The differential equation $(3\,a^2x^2 + by\cos x)dx + (2\sin x - 4ay^3)dy = 0$ is exact for:

a) $a = 3, b = 2$
b) $a = 3, b = 4$
c) $a = 2, b = 3$
d) $a = 2, b = 5$

Q.20 Let G be a cyclic group of order 6. Then, the no. of elements $g \in G$ such that $G = < g >$ is:

a) 2
b) 3
c) 4
d) 5

Q.21 One particular solution of $y''' - y'' - y' + y = e^x$ is a constant multiple of:

a) xe^{-x}
b) x^2e^{-x}
c) xe^x
d) x^2e^x

Q.22 Consider the wave equation

$\dfrac{\partial^2 u}{\partial t^2} = 4\dfrac{\partial^2 u}{\partial x^2}, 0 < x < \pi, t > 0$, with

$u(0,t) = u(\pi,t) = 0, u(x,0) = 3\sin x$ and

$\dfrac{\partial u}{\partial t} = 0$ at $t = 0$. Then, $u\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$ is:

a) 2
b) 1
c) 0
d) -3

Q.23 The number of 5-Sylow subgroups of Z_{20} is:

a) 1
b) 4
c) 5
d) 6

Q.24 The integrating factor of $(y^3 - 2x^2y)dx + (2xy^2 - x^3)dy = 0$ is $x^m y^n$, then

a) $m = -7, n = 1$
b) $m = 1, n = 1$
c) $m = 7, n = 1$
d) $m = -1, n = 7$

Q.25 The differential equation whose linear independent solutions are $\cos x,\ \sin x,\ e^{-x}$ is

a) $(D^3 + D^2 + D + 1)y = 0$
b) $(D^3 - 4D^2 + 4D + 4)y = 0$
c) $(D^3 - D^2 + D - 1)y = 0$
d) $(D^3 + D^2 - D + 1)y = 0$

where $D = \dfrac{dy}{dx}$

Q.26 The eigen values of the Sturm Liouville system $y'' + \lambda y = 0, 0 \le x \le \lambda,\ \ y(0) = 0$, $y'(\pi) = 0$ are

a) $\dfrac{n^2}{4}$

b) $\dfrac{(2n-1)^2\pi^2}{4}$

c) $\dfrac{(2n+1)^2}{4}$

d) $\dfrac{n^2\pi^2}{4}$

Q.27 Let G be a finite group of order 200. Then, the no. of subgroups of G of order 25 is:

a) 1
b) 4
c) 5
d) 10

Q.28 The general integral of the partial differential equation

$(y+z)\frac{\partial z}{\partial x} - (x+z)\frac{\partial z}{\partial y} = x - y$ is:

a) $F(x+y+z, x^2+y^2-z^2) = 0$
b) $F(x-y+z, x^2+y^2-z^2) = 0$
c) $F(x+y+z, x^2-y^2-z^2) = 0$
d) $F(x+y-z, x^2-y^2-z^2) = 0$

Q.29 Let $A = \{1,2,3\}$ and let

$R_1 = \{(1,1), (1,3), (3,1), (2,2), (2,1), (3,3)\}$

$R_2 = \{(1,3), (3,1), (2,2)\}$

$R_3 = \{(1,3), (3,3)\}$

$R_4 = A \times A$

Then the relation R_4 on A is:

a) Reflexive
b) Symmetric
c) Transitive
d) All of the above

Q.30

$$2\frac{\partial^2 z}{\partial x^2} + 5\frac{\partial^2 z}{\partial x \partial y} + 3\frac{\partial^2 z}{\partial y^2} = 0$$

The partial differential equation is:

a) Elliptic
b) Parabolic
c) Hyperbolic
d) None

Q.31 For what values of α and β, the quadratic formula

$$\int_{-1}^{1} f(x)dx = \alpha f(-1) + f(\beta)$$

is exact for all polynomials of degree ≤ 1?

a) $\alpha = 1, \beta = 1$
b) $\alpha = 1, \beta = -1$
c) $\alpha = -1, \beta = 1$
d) $\alpha = -1, \beta = -1$

Q.32 The eigenvalue λ of integral equation

$$y(x) = \lambda \int_0^1 x^2 t^2 y(t)dt$$

is:

a) $\lambda = 5$
b) $\lambda = -5$
c) $\lambda = 2$
d) $\lambda = -2$

Q.33 The differential equation corresponding to the integral equation

$$y(x) = \int_0^x t(t-x)y(t)dt + \frac{1}{2}x^2$$

is:

a) $y'' + xy = 1, y(0) = y'(0) = 0$
b) $y'' + y = 1, y(0) = y'(0) = 0$
c) $y'' + 5xy = 1, y(0) = y'(0) = 0$
d) None of these

Q.34 The integral equation

$$y(x) = 1 + x + \int_0^x (x-t)y(t)dt$$

is solved by the method of successive approximations. Starting with initial approximation $y_0(x) = 1 + x$, the second approximation, $y_2(x)$, is given by:

a) $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!}$
b) $x + \frac{x^3}{3!} + \frac{x^5}{5!}$
c) $1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} - \frac{x^5}{5!}$

d) $1+\frac{x^2}{2!}+\frac{x^4}{4!}$

Q.35 If $g:[1,2]\to R$ is a non-negative Riemann-integrable function such that

$$\int_1^2 \frac{g(x)}{x^4}dx = k\int_1^2 g(x)dx \neq 0$$

Then k belongs to the interval

a) $\left[0,\frac{1}{16}\right]$
b) $\left[\frac{1}{16},1\right]$
c) $\left[\frac{1}{16},0\right]$
d) None of these

Q.36 Let $g:[0,\infty)\to R$ be defined by

$$g(x)=\begin{cases}\frac{1}{\sqrt{x}}, x\neq 0\\ 0, x=0\end{cases}$$

Consider the two improper integral

$$I_1=\int_0^1 g(x)dx \text{ and } I_2=\int_1^\infty g(x)dx$$

Then,

a) Both I_1 and I_2 exist
b) I_1 exists but I_2 does not
c) I_2 exists but I_1 does not
d) Neither I_1 nor I_2 exists

Q.37 The continuous function $g:R\to R$ defined by $g(x)=(x^2+1)^{501}$ is:

a) Onto but not one-one
b) One-one but not onto
c) Both one-one and onto
d) Neither one-one nor onto

Q.38 The series

$$\sum_{n=1}^{\infty}\frac{\cos nx}{n^{5/2}}, \forall x\in R$$

is:

a) Uniformly but not absolutely convergent
b) Uniformly and absolutely convergent
c) Neither absolutely nor uniformly convergent
d) Absolutely but not uniformly convergent

Q.39 The connected subsets of the real line with the usual topology are:

a) All intervals
b) Only compact intervals
c) Only bounded intervals
d) Only semi-infinite intervals

Q.40 A uniform thin bar of mass $6m$ and length $12L$ is bent to make a regular hexagon. Its moment of inertia about an axis passing through the centre of mass and perpendicular to the plane of hexagon is:

a) $20mL^2$
b) $6mL^2$
c) $(12/5)mL^2$
d) $30mL^2$

Mathematics

PLD - 2

1. B
2. B
3. B
4. A
5. A
6. A
7. C
8. D
9. C
10. A
11. A
12. C
13. B
14. B
15. D
16. D
17. D
18. A
19. A
20. A
21. D
22. D
23. A
24. B
25. A
26. C
27. A
28. A
29. D
30. C
31. A
32. A
33. A
34. A
35. B
36. B
37. D
38. B
39. A
40. A