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PhD ENTRANCE TEST (PET) -MATHEMATICS

Time duration: 45 Minutes **Maximum Marks**:

1. The initial value problem: $\left(x^2 - x\right)\frac{dy}{dx} = (2x-1)y,\ \ y(x_0) = y_0$ has a unique solution if (x_0, y_0) equals

   (A) $(2, 1)$ (B) $(0,\ 0)$ (C) $(0,\ 1)$ (D) $(1, 1)$

2. Linear combinations of solutions of an ordinary differential equation are also solutions if the differential equation is

   (A) Linear homogeneous (B) Linear non-homogeneous

   (C) Nonlinear homogeneous (D) Nonlinear non-homogeneous

3. Let n be a non-negative integer. The eigenvalues of the Sturm-Liouville problem:

   $\frac{d^2y}{dx^2} + \text{a}\ y = 0,$ with boundary conditions $y(0) = y(2\circledR),\ \frac{dy}{dx}(0) = \frac{dy}{dx}(2\circledR)$ are

   (A) n^2 (B) $n^2\circledR^2$ (C) $n\circledR$ (D) n

4. The solution of the partial differential equation: $x\,p + y\,q = z$ is

   (A) $f(x, y) = 0$ (B) $f\left(\frac{x}{y}, \frac{y}{z}\right) = 0$ (C) $f(x\,y,\ y\,z) = 0$ (D) $f\left(x^2,\ y^2\right) = 0$

5. Which of the following is elliptic?

   (A) Wave equation (B) Laplace equation

   (C) Heat equation (D) $u_{xx} + 2u_{xy} - 4u_{yy} = 0$

6. The solution of the Cauchy problem: $u_{yy}(x, y) - u_{xx}(x, y) = 0;$ $u(x,0) = 0,\ u_y(x,\ 0) = x$ is

   (A) $x\,y$ (B) $\frac{x}{y}$ (C) $x\,y + \frac{x}{y}$ (D) 0

7. In three dimensions, the equation: $x^2 - y^2 = a^2$ represents

(A) Sphere (B) Pair of straight lines (C) Cone (D) A cylinder

8. The gradient of a scalar field¤ $(x, y) = y^2 - 4xy$ at $(1, 2)$ is

(A) $8\hat{i}$ (B) $-8\hat{i}$ (C) $-8\hat{j}$ (D) $8\hat{j}$

9. The value of the line integral $\int_C \left(y^2\,dx + x^2\,dy\right)$, where C is the boundary of the square: $-1 \le x \le 1,\ -1 \le y \le 1$ is equal to

(A) 4 (B) 0 (C) $2(x+y)$ (D) $\frac{4}{3}$

10. A necessary and sufficient condition for the line integral $\int_C \vec{F}.d\vec{r} = 0$ for very closed curve C is that

(A) $\vec{\nabla}.\vec{F} = 0$ (B) $\vec{\nabla}.\vec{F} \neq 0$ (C) $\vec{\nabla} \times \vec{F} = 0$ (D) $\vec{\nabla} \times \vec{F} \neq 0$

11. The Laplace transform of the unit step function: $u(t-a) = \begin{cases} 0, & t < a \\ 1, & t \ge a \end{cases}$ is equal to

(A) $\frac{e^{-as}}{s}$ (B) $\frac{e^{as}}{s}$ (C) $a\,e^{-as}$ (D) $s\,e^{-as}$

12. If $f(s)$ is the Laplace transform of $f(t)$, then Laplace transform of $\int_0^t f(t)\,dt$ is

(A) $\frac{1}{s} f(s)$ (B) $\frac{1}{s} f(s) - f(0)$ (C) $s\,f(s) - f(0)$ (D) $\int f(s)\,ds$

13. The Fourier transform of a function $f(x)$ is $F(\mu)$. Then Fourier transform of $f'(x)$ will be

(A) 2® $i\,\mu\,F(\mu)$ (B) $i\,\mu\,F(\mu)$ (C) $\frac{d\,F(\mu)}{i\,\mu}$ (D) $\frac{d\,F(\mu)}{d\mu}$

14. Newton iterative formula to find the value of $\sqrt{N}$ is

(A) $x_{n+1} = \frac{1}{2}\left(x_n + \frac{1}{N\, x_n} \right)$ (B) $x_{n+1} = x_n \; \left(2 - N\, x_n\right)$

(C) $x_{n+1} = \frac{1}{2}\left(x_n + \frac{N}{x_n} \right)$ (D) $x_{n+1} = \frac{{x_n}^2 + N}{x_n}$

15. As soon as a new value of a variable is found by iteration for solving $AX = B$, it is used immediately in the following equations, this method is called

(A) Gauss elimination method (B) Gauss-Jordan method

(C) Gauss-Seidel method (D) Jacobi method

16. The 100th divided difference of a polynomial of degree 100 is

(A) 0 (B) 1 (C) a constant (D) variable

17. Using Picard's method to solve: $\frac{dy}{dx} = 1 - 2\,x\,y,\;\; y(0) = 0$, the first approximation $y_1(x)$ is

(A) $x - \frac{2}{3}x^3$ (B) x (C) x^2 (D) $x + \frac{2}{3}x^2$

18. The solution of the integral equation $¤\left(x\right) = x + \int_0^x (¶ - x)\, ¤(¶)\, d¶$ is

(A) Sec x (B) Sin x (C) Cos x (D) Tan x

19. Given an integral equation: $¤\left(x\right) = \frac{1}{e^2 - 1}\int_0^1 2\,e^x\; e^{¶}\; ¤(¶)\, d¶$. Then

(A) The solution of the integral equation is $¤\left(x\right) = 1$.

(B) The solution of the integral equation is $¤\left(x\right) = \cos x$.

(C) The equation is Freedholm integral equation

(D) The equation is Volterra integral equation

20. If the Lagrangian does not depend on time explicitly, then

(A) The Hamiltonian is constant (B) The Hamiltonian can not be constant

(C) The kinetic energy is constant (D) The potential energy is constant

21. The product of generalized coordinate and conjugate momentum has the dimension of

(A) Energy (B) Force (C) Linear momentum (D) Angular momentum

22. The value of k for which the given set of vectors $\{(1, k, 5), (1, -3, 2), (2, -1, 1)\}$ forms a basis of R^3 is

(A) $k \neq 0$ (B) $k \neq 1$ (C) $k \neq -6$ (D) $k \neq -8$

23. Dimension of the subspace W of R^3 defined by $W = \{(a, b, c) : a + b + c = 0, a = b = c\}$ is

(A) 0 (B) 1 (C) 2 (D) 3

24. Suppose that a is an eigenvalue of a non-singular square matrix M. Then

(A) $|M|$ is an eigenvalue of $adj(M)$ (B) $\frac{1}{a}$ is an eigenvalue of $adj(M)$

(C) 0 is an eigenvalue of $adj(M)$ (D) $\frac{|M|}{a}$ is an eigenvalue of $adj(M)$

25. Let $a, b \in G$, where G is a group. Which of the following is not true in general?

(A) $(a^{-1})^{-1} = a$ (B) $(ab)^{-1} = b^{-1} a^{-1}$

(C) $(ab)^2 = a^2 b^2$ (D) Each $a \in G$ has a unique inverse

26. Let G be a group of order 21. Then which of the following is not correct?

(A) G has exactly three Sylow 3 subgroups

(B) G has one or seven Sylow 3 subgroups

(C) G has a Sylow 7 subgroups

(D) Sylow 7 subgroups of G is normal.

27. Let R be a ring. If $R[x]$ is a principal ideal domain, then R is necessarily

(A) A field (B) An Euclidean domain

(C) A unique factorization domain (D) A principal ideal domain

28. Let (X, d) be a metric space. Which of the following is possible?

(A) X has exactly 3 dense subsets (B) X has exactly 5 dense subsets

(C) X has exactly 6 dense subsets (D) X has exactly 4 dense subsets

29. Let X be the set of all irrational numbers with discrete metric. Then which of the following is true?

(A) X is complete (B) X is compact

(C) X is connected (D) X is bounded

30. Let X be a metric space. Let $A \subseteq X$. Then which of the following statements is true?

(A) If A is compact in X, then A is closed in X.

(B) If A is closed in X, then A is compact in X.

(C) If $A \subseteq X$ is open, then A has no limit point.

(D) If A has an empty interior, then all points of A are isolated.

31. Let X, Y be topological spaces and $f : X \to Y$ be a continuous and bijective map. Then f is a homomorphism, if

(A) X and Y are compact (B) X is Hausdroff and Y is compact

(C) X is compact and Y is Hausdroff (D) X and Y are Hausdroff

32. Let $a_n = \begin{cases} 1, \; if \;\; n = 3k, \; k = 1, \, 2, \, 3, --- \\ \dfrac{1}{(n+1)}, \; if \;\; n = 3k-2, \; k = 1, \, 2, 3, --. \\ \dfrac{2}{(n+1)}, \; if \;\; n = 3k-1, \; k = 1, \, 2, 3, -- \end{cases}$ Then

(A) $\limsup a_n = 1, \; \liminf a_n = 0$ (B) $\limsup a_n = 1, \; \liminf a_n = -\infty$

(C) $\limsup a_n = \infty, \; \liminf a_n = 0$ (D) $\limsup a_n$ and $\liminf a_n$ does not exist

33. The improper integral $\int_{-\infty}^{0} 2^x \, dx$

(A) Convergent and converges to 2 (B) Divergent

(C) Convergent and converges to $\frac{1}{\ln 2}$ (D) Convergent and converges to $-\ln 2$.

34. Let Q denotes the set of rational in R. Let m denotes the Lebesgue measure in R. Then

(A) $m(Q)=1$ (B) $m(Q)=2$ (C) $m(Q)=0$ (D) $m(Q)=\infty$

35. For $f(z)=\frac{z-\sin z}{z^3}$, the point $z=0$ is

(A) A pole (B) Removable singularity

(C) Essential singularity (D) Zero of order 1

36. The value of $\int_C \frac{z^2+1}{z^2-1}dz$, where C is the circle with centre at $z=1$ and radius unity is

(A) -2® (B) ® i (C) 2® i (D) 4® i

37. The mapping $w=z^2$ maps the first quadrant onto

(A) Itself (B) Upper half plane (C) Third quadrant (D) Right half plane

38. Suppose that P and Q are independent events of an experiment E. Then it follows that

(A) P^c and Q^c are dependent (B) P and Q are mutually exclusive

(C) P and Q^c are dependent (D) P^c and Q are independent

39. Let $P(X=n)=\frac{ª}{n^2\,(n+1)}$, where ª is an appropriate constant. Then $E(X)$ is

(A) 2ª $+1$ (B) ª (C) ∞ (D) 2ª

40. Let b_{yx} and b_{xy} denote the regression coefficients of Y on X and of X on Y respectively. Then $b_{yx}=b_{xy}$ implies that

(A) $±_y = ±_x$ (B) $° = 0$ (C) $° = 1$ (D) $±_y \neq ±_x$

PhD Entrance TEST PET-MATHEMATICs

ANSWER KEY

Q. No.	**Answer**	**Q. No.**	**Answer**
1.	**(D)**	21.	**(D)**
2.	**(A)**	22.	**(D)**
3.	**(A)**	23.	**(A)**
4.	**(B)**	24.	**(D)**
5.	**(B)**	25.	**(C)**
6.	**(A)**	26.	**(A)**
7.	**(D)**	27.	**(A)**
8.	**(B)**	28.	**(D)**
9.	**(B)**	29.	**(D)**
10.	**(C)**	30.	**(A)**
11.	**(A)**	31.	**(C)**
12.	**(A)**	32.	**(A)**
13.	**(B)**	33.	**(C)**
14.	**(C)**	34.	**(C)**
15.	**(C)**	35.	**(B)**
16.	**(C)**	36.	**(C)**
17.	**(B)**	37.	**(B)**
18.	**(B)**	38.	**(D)**
19.	**(C)**	39.	**(B)**
20.	**(A)**	40.	**(A)**